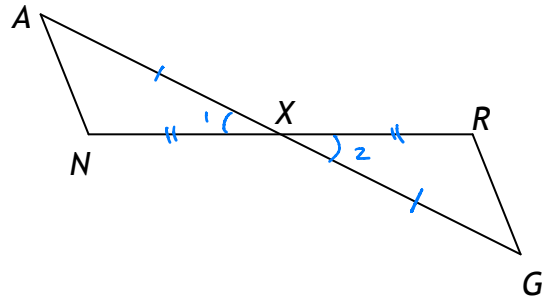


# Proving Congruent Triangles

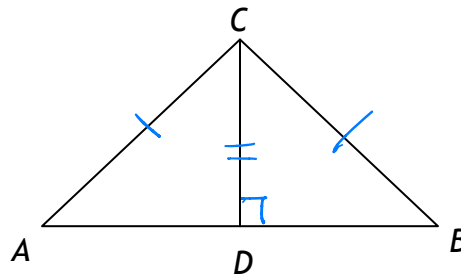
1. **Given:**  $X$  is midpoint of  $\overline{AG}$  and  $\overline{NR}$   
**Prove:**  $\triangle ANX \cong \triangle GRX$



Statements	Reasons
⑤ $\overline{AX} = \overline{GX}$	Def midpt
⑥ $\angle 1 \cong \angle 2$	Vert $\angle$ 's $\cong$ Thm
⑤ $\overline{NX} = \overline{XR}$	Def mid point
$\triangle ANX \cong \triangle GRX$	SAS

2. **Given:**  $\overline{AC} \cong \overline{BC}$ ,  $\overline{CD} \perp \overline{AB}$   
**Prove:**  $\triangle ACD \cong \triangle BCD$

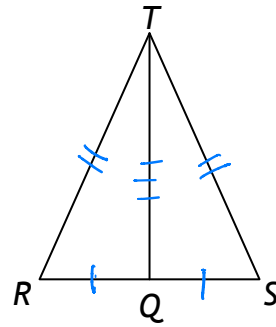
Plan use HL



Statements	Reasons

3. **Given:**  $\overline{TQ}$  is the median of  $\triangle TRS$ ,  $\overline{TR} \cong \overline{TS}$

**Prove:**  $\angle TRQ \cong \angle TSQ$



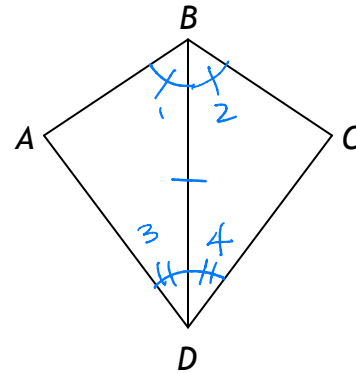
Plan SSS

Statements	Reasons

4. **Given:**  $\overline{BD}$  bisects  $\angle ABC$ , and  $\angle ADC$

**Prove:**  $\triangle ABD \cong \triangle CBD$

Plan: Use ASA

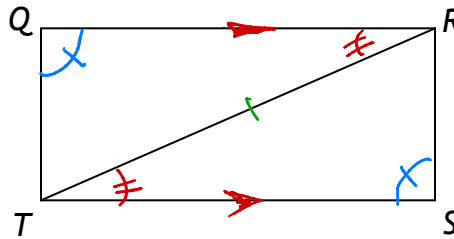


Statements	Reasons

5.

**Given:**  $\angle TQR \cong \angle RST$ ,  $\overline{QR} \parallel \overline{TS}$

**Prove:**  $\triangle QRT \cong \triangle STR$



Plan: AAS

Statements	Reasons

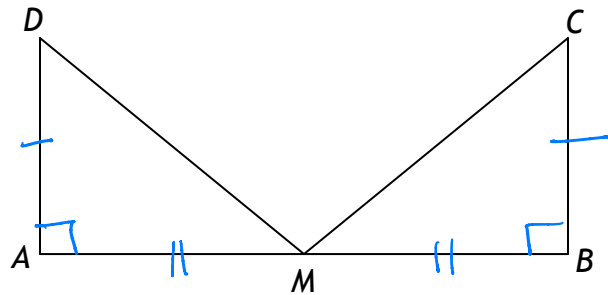
6.

**Given:**  $\overline{DA} \perp \overline{AB}$ ,  $\overline{CB} \perp \overline{AB}$ ,  $\overline{DA} \cong \overline{CB}$ ,

$M$  is the midpoint of  $\overline{AB}$

**Prove:**  $\triangle ADM \cong \triangle BCM$

Plan: SAS

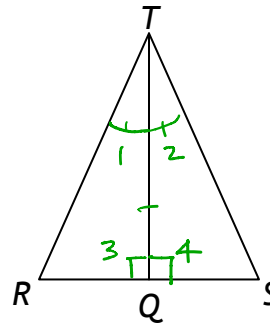


Statements	Reasons

Introducing a new definition!

Since we are proving two triangles congruent, then it follows that their corresponding parts are congruent. We typically abbreviate this in a proof using **CPCTC** which stands for:

7. **Given:**  $\overline{TQ}$  bisects  $\angle RTS$ ,  $\overline{TQ} \perp \overline{RS}$   
**Prove:**  $\overline{RQ} \cong \overline{SQ}$

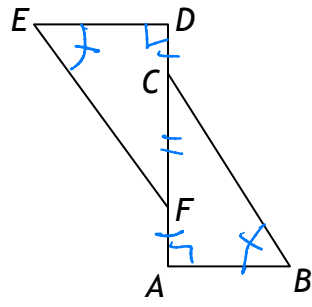


Plan:  
ASA

Statements	Reasons
① $\angle 1 \cong \angle 2$	Def. $\angle$ bisector
⑤ $\overline{TQ} \cong \overline{TQ}$	Reflexive
① $\angle 3 \cong \angle 4$	Def of $\perp$
$\triangle RQT \cong \triangle SQT$	ASA
$\overline{RQ} \cong \overline{SQ}$	CPCT

8. **Given:**  $\angle FED \cong \angle CBA$ ,  $\overline{DC} \cong \overline{AF}$ ,  $\overline{FD} \perp \overline{DE}$ ,  $\overline{CA} \perp \overline{AB}$   
**Prove:**  $\overline{EF} \cong \overline{BC}$

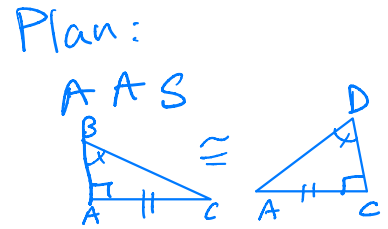
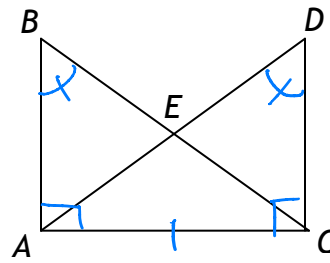
Plan:  $\triangle FED \cong \triangle CBA$  (AAS)  
 if  $\overline{DF} \cong \overline{AC}$ , why?



Statements	Reasons



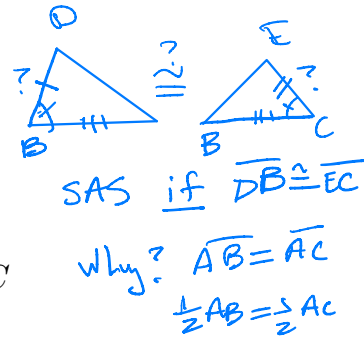
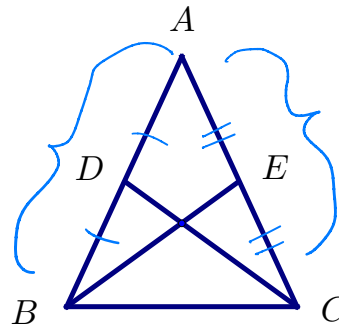
11. **Given:**  $\angle B \cong \angle D$ ,  $\overline{AC} \perp \overline{BA}$ ,  $\overline{DC} \perp \overline{AC}$   
**Prove:**  $\overline{AB} \cong \overline{CD}$



Statements

Reasons

12. **Given:**  $\overline{AB} \cong \overline{AC}$ ,  $\overline{CD}$  bisects  $\overline{AB}$ ,  $\overline{BE}$  bisects  $\overline{AC}$   
**Prove:**  $\overline{CD} \cong \overline{BE}$



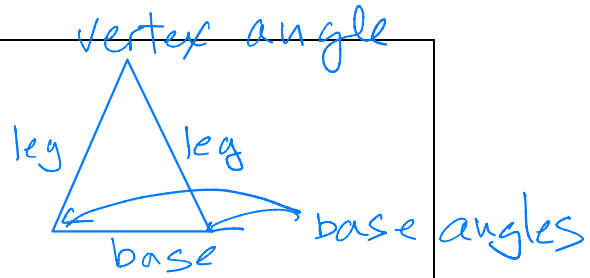
Statements

Reasons

# Time to Step it Up a Notch!!

## Isosceles Triangle Theorem

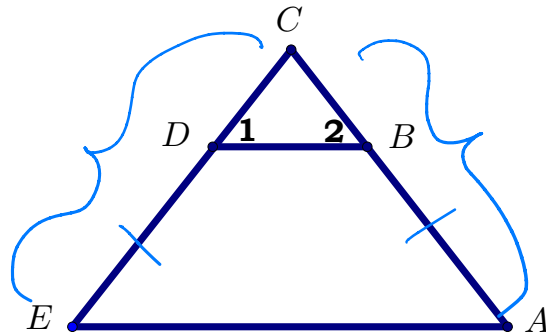
If base angle  $\cong$ , legs  $\cong$



## Isosceles Triangle Theorem - Corollary

If legs  $\cong$ , base angles  $\cong$

13. **Given:**  $\overline{CA} \cong \overline{CE}$ ,  $\overline{BA} \cong \overline{DE}$   
**Prove:**  $\angle 1 \cong \angle 2$



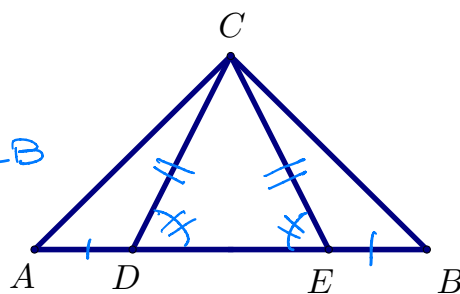
Statements	Reasons
1. $\overline{CA} \cong \overline{CE}$	1. Given
2. $\overline{CA} \cong \overline{CB} + \overline{BA}$ $\overline{CE} \cong \overline{CD} + \overline{DE}$	2. Seg Addition
3. $\overline{CB} + \overline{BA} \cong \overline{CD} + \overline{DE}$	3. Tran
4. $\overline{BA} \cong \overline{DE}$	4. given
⑤ 5. $\overline{CB} \cong \overline{CD}$	5. Subtraction (line 4 from ln 3)
6. $\angle 1 \cong \angle 2$	6. Isosc. $\Delta$ Thm (corollary)



14. **Given:**  $\overline{AD} \cong \overline{BE}$ ,  $\overline{CD} \cong \overline{CE}$ ,  $\overline{ADEB}$

**Prove:**  $\overline{AC} \cong \overline{BC}$

Plan: need to show  $\angle A \cong \angle B$   
by first proving  
 $\triangle ADC \cong \triangle BEC$   
(SAS)

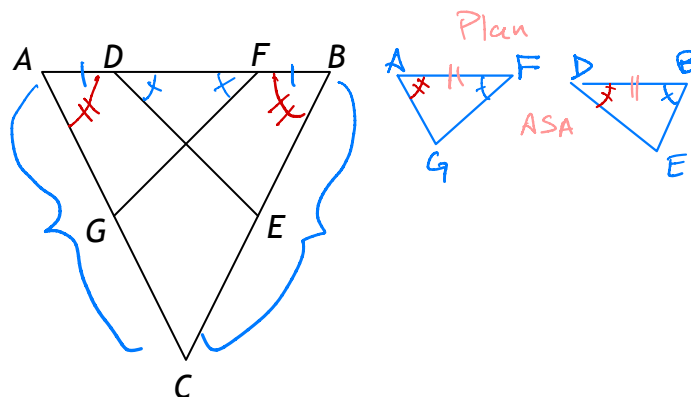


Statements

Reasons

15. **Given:**  $\overline{AD} \cong \overline{FB}$ ,  $\angle BDE \cong \angle AFG$   $\overline{AC} \cong \overline{BC}$

**Prove:**  $\triangle AFG \cong \triangle BDE$



Statements

Reasons

1.  $\overline{AD} \cong \overline{FB}$

2.  $\overline{DF} \cong \overline{DF}$

3.  $\overline{AD} + \overline{DF} \cong \overline{FB} + \overline{DF}$

4.  $\overline{AF} \cong \overline{AD} + \overline{DF}$   
 $\overline{DB} \cong \overline{DF} + \overline{FB}$

(S) 5.  $\overline{AF} = \overline{DB}$

(A) 6.  $\angle BDE \cong \angle AFG$

(A) 7.  $\angle FDE \cong \angle DFG$

8.  $\triangle AFG \cong \triangle BDE$

1. given

2. Reflexive

3. Addition (Steps 1 & 2)

4. Seg Add

5. Trans (or Subst)

6. Given

7. Isosc. A Thm (corr.)

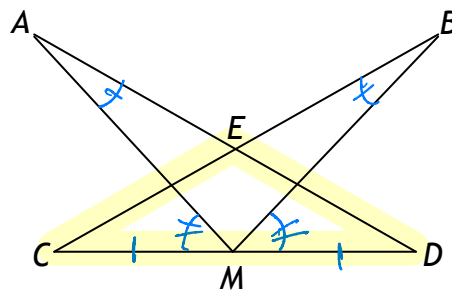
8. ASA

16. **Given:**  $\angle MAD \cong \angle MBC$ ,  $\angle AMC \cong \angle BMD$

$M$  is the midpoint of  $\overline{DC}$

**Prove:**  $\triangle CED$  is isosceles

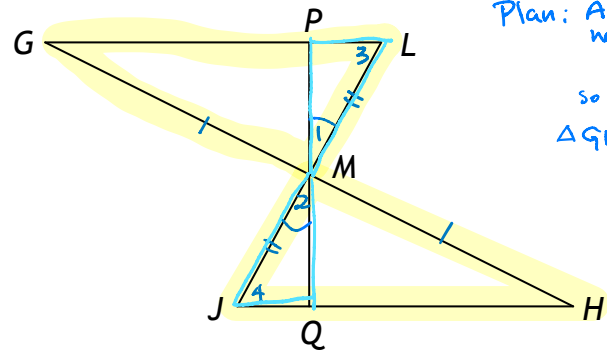
Plan: need to show  $\angle C \cong \angle D$   
by proving  $\triangle CMB \cong \triangle DMA$   
(AAS)



Statements

Reasons

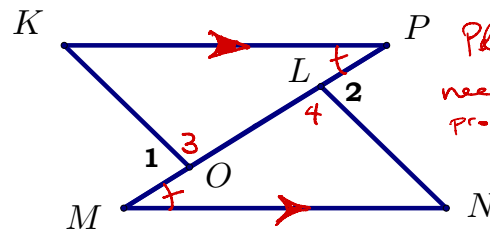
17. **Given:**  $\overline{GH}$  and  $\overline{JL}$  bisect each other  
**Prove:**  $\triangle JQM \cong \triangle LPM$



Plan: ASA  
 needs  $\angle 3 \cong \angle 4$ ,  
 so prove  
 $\triangle GML \cong \triangle HMT$   
 first

Statements	Reasons
(S) $\overline{GM} \cong \overline{HM}$ (A) $\angle GML \cong \angle HMJ$ (S) (S) $\overline{LM} \cong \overline{JM}$ $\triangle GML \cong \triangle HMJ$ (A) $\angle 3 \cong \angle 4$ (A) $\angle 1 \cong \angle 2$ $\triangle JQM \cong \triangle LPM$	Def bisector Vert $\angle$ 's $\cong$ Thm Def bisector SAS CPCT Vert $\angle$ 's $\cong$ Thm ASA

18. **Given:**  $\overline{KP} \parallel \overline{MN}$ ,  $\overline{ML} \cong \overline{PO}$ ,  $\angle 1 \cong \angle 2$   
**Prove:**  $\triangle MON \cong \triangle PLK$



Plan ASA,  
 need to first  
 prove  $\angle 3 \cong \angle 4$

Statements	Reasons

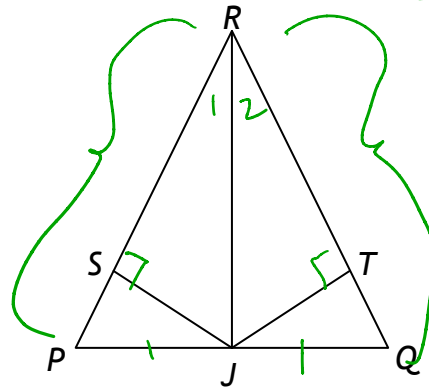
19. **Given:**  $\overline{SJ} \perp \overline{PR}$ ,  $\overline{TJ} \perp \overline{RQ}$ ,  $\overline{RP} \cong \overline{RQ}$

$J$  is the midpoint of  $\overline{PQ}$

**Prove:**  $\triangle SJR \cong \triangle TJR$

Plan:  $\triangle SJR \cong \triangle TJR$  by AAS  
if  $\angle 1 = \angle 2$ ,

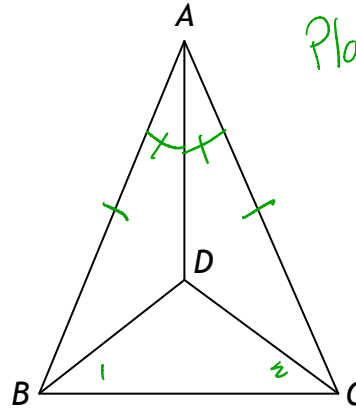
So prove  
 $\triangle PJR \cong \triangle QJR$   
by SSS



Statements

Reasons

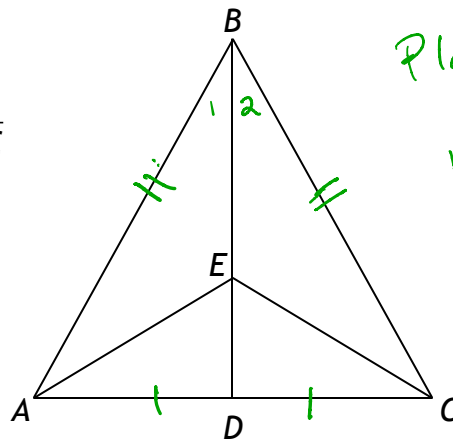
20. **Given:**  $\overline{AB} \cong \overline{AC}$ ,  $\overline{AD}$  bisects  $\angle BAC$   
**Prove:**  $\angle DBC \cong \angle DCB$   
 ( $\angle 1 \cong \angle 2$ )



Plan:  $\triangle ABC$  is  
 Isosceles if  
 $\overline{BD} \cong \overline{DC}$   
 so first  
 prove  
 $\triangle ADB \cong \triangle ADC$

Statements	Reasons

21. **Given:**  $\angle BAD \cong \angle BCD$ ,  $D$  is the midpoint of  $\overline{AC}$   
**Prove:**  $\overline{AE} \cong \overline{CE}$



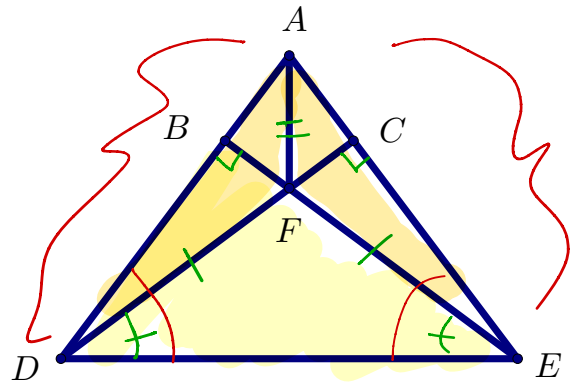
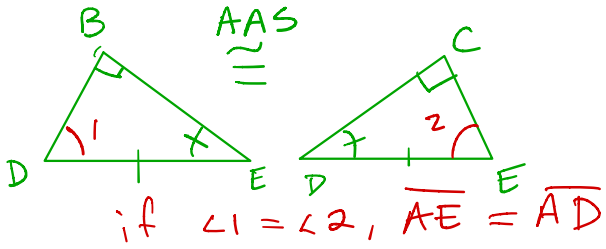
Plan:  
 prove  $\triangle BEA \cong \triangle BEC$   
 (SAS)  
 by showing  $\angle 1 = \angle 2$   
 as it is a  
 corr part of  
 $\triangle BAD \cong \triangle BCD$   
 (SSS)

Statements	Reasons

Plan: prove  $\triangle AFE \cong \triangle AFD$  by SSS

22. **Given:**  $\overline{BE} \perp \overline{AD}$ ,  $\overline{CD} \perp \overline{AE}$ ,  $\overline{EF} \cong \overline{DF}$ ,  
**Prove:**  $\triangle AFE \cong \triangle AFD$

Plan

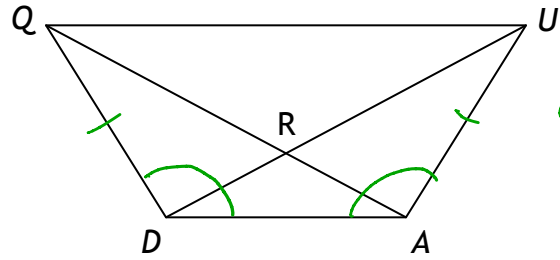


Statements

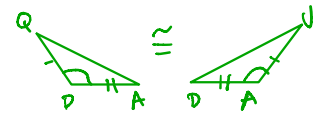
Reasons

Statements	Reasons

23. **Given:**  $\angle QDA \cong \angle UAD, \overline{QD} \cong \overline{UA}$   
**Prove:**  $\triangle QDR \cong \triangle UAR$



Plan: SAS



Statements	Reasons



24. **Given:**  $\overline{GI} \cong \overline{JH}$ ,  $\overline{GH} \perp \overline{HI}$ ,  $\overline{JI} \perp \overline{HI}$

**Prove:**  $\overline{GK} \cong \overline{JK}$

Plan

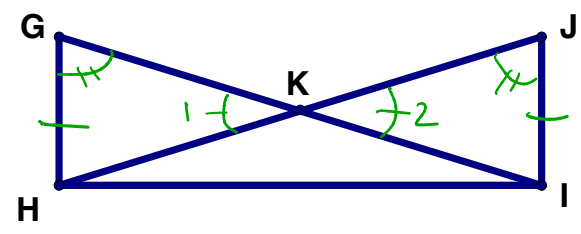


So  $\angle G = \angle J$

$\overline{GH} \cong \overline{JI}$

$\Delta GKH \cong \Delta JKI$

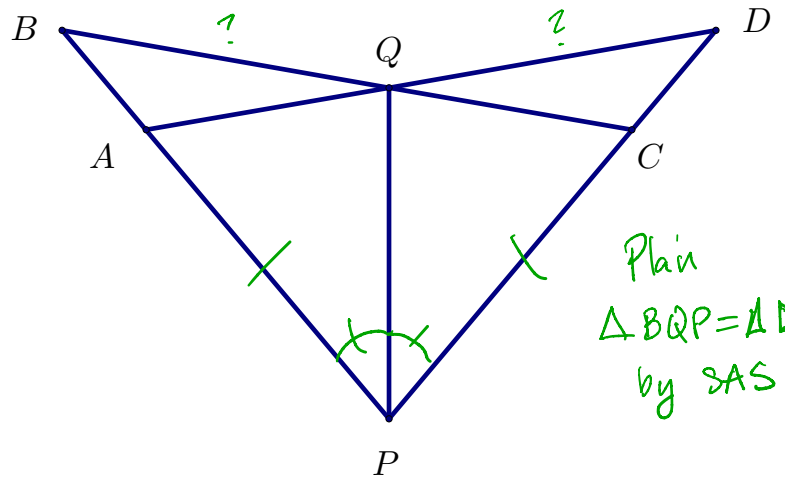
AAS



Statements	Reasons
1. $\overline{GI} \cong \overline{JH}$ , $\overline{GH} \perp \overline{HI}$ $\overline{JI} \perp \overline{HI}$	1. given
2. $\overline{HI} = \overline{HI}$	2. Reflexive
3. $\Delta GHI \cong \Delta JIH$	3. HL
4. $\angle G \cong \angle J$ $\overline{GH} \cong \overline{JI}$	4. CPCT $\cong$
5. $\angle 1 \cong \angle 2$	5. Vert $\angle$ 's $\cong$ Thm
6. $\Delta GKH \cong \Delta JKI$	6. AAS
7. $\overline{GK} \cong \overline{JK}$	7. CPCT

25. **Given:**  $\angle BPQ \cong \angle DPQ$ ,  $\overline{AP} \cong \overline{CP}$

**Prove:**  $\overline{QB} \cong \overline{QD}$

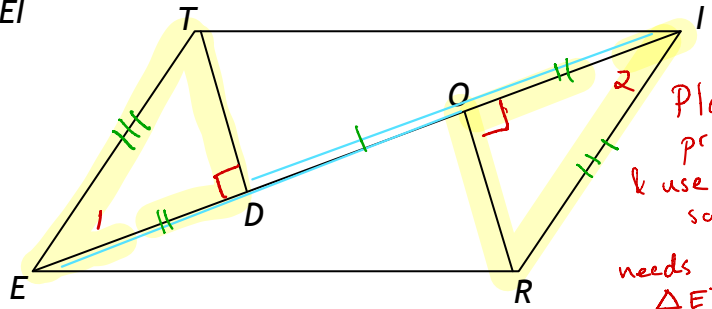


Statements

Reasons

26. **Given:**  $\overline{EO} \cong \overline{ID}$ ,  $\overline{ET} \cong \overline{IR}$ ,  $\overline{TD} \perp \overline{EI}$ ,  $\overline{RO} \perp \overline{EI}$

**Prove:**  $\overline{TI} \parallel \overline{RE}$



Plan:  
 prove  $\angle 1 = \angle 2$   
 & use AI to  
 say  $\overline{TI} \parallel \overline{RE}$   
 needs  
 $\triangle EDT \cong \triangle IOR$   
 (HL)

Statements

Reasons

1.  $\overline{EO} \cong \overline{ID}$
2.  $\overline{EO} \cong \overline{ED} + \overline{DO}$   
 $\overline{ID} \cong \overline{DO} + \overline{OI}$
3.  $\overline{ED} + \overline{DO} = \overline{DO} + \overline{OI}$
4.  $\overline{DO} \cong \overline{DO}$
5.  $\overline{ED} \cong \overline{OI}$
6.  $\overline{ET} = \overline{IR}$
7.  $m\angle 3 = m\angle 4 = 90^\circ$
8.  $\triangle EDT = \triangle IOR$
9.  $\angle 3 = \angle 4$
10.  $\overline{TI} \parallel \overline{RE}$

1. Given
2. Seg Add
3. Trans.
4. Reflexive
5. Subtraction (start 4 from stat 3)
6. Given
7. Def of  $\perp$
8. HL
9. CPCT
10. If Alt. Int  $\angle$ 's  $\cong$ ,  
 the lines are  $\parallel$

(L)  
 (H)  
 (H)